A Realistic Superstring-Inspired Brane-World Cosmology

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- What are Brane-Worlds?
  - Some history, and what Brane-Worlds are (not)

- What are Stringy Brane-Worlds?
  - (Super)strings and Cosmology interplay

- What are “ET”s, and Why Censor Them?
  - A potentially disastrous observation & its cure

- What is Dynamical Censorship?
  - Dynamics of matter around brane-Worlds and its consequences
What Brane-Worlds are **not**...

1914: Gunnar Nordström, (early competitor of Einstein’s relativity)
1919: Theodor F.E. Kałuża
1926: Oscar Klein

Our spacetime is a sub-spacetime of a bigger (5-dimensional) one

- **The metric tensor**
  \[ ds^2 = g_{ij} \, dx^i \, dx^j, \text{ where } i,j = 0,1,2,3,4 \]

  decomposes: \((g_{\mu\nu}, g_{\mu4}, g_{44})\) where
  - \(g_{04}\) plays the role of \(\Phi\),
  - \((g_{14}, g_{24}, g_{34})\) play the role of \(\mathbf{A}\),
  - ...of electromagnetism
  - Einstein equations beget Maxwell’s
  - provided \(x^4\) curls up into a circle:
What Brane-Worlds are not...

(Nordström)-Kałuża-Klein compactification:

“extra” dim’s are curled up (compact), \( \leq 10^{-33} \text{m} \);

fields decompose into “Fourier modes” over extra dimensions

- Only 1 extra dimension \( \bowtie \) easy: \( e^{in \cdot x^4 / (2\pi R)} \),

- The mass (inertia) of the \( n^{th} \) mode = \( n \cdot (\hbar / 2\pi c R) \geq 10^{17} \text{ GeV}/c^2 \),
  that is, \( \Box_5 \rightarrow \Box_4 + [n \cdot (1/2\pi R)]^2 \);

- Plus the \( g_{ij} \Rightarrow (g_{\mu\nu}, g_{\mu4}, g_{44}) \) decomposition:

\[
\begin{align*}
g^{7}_{\mu \nu}(x^0,\ldots,x^3), & \quad g^{7}_{\mu 4}(x^0,\ldots,x^3), \quad g^{7}_{44}(x^0,\ldots,x^3) & \sim 7 \times 10^{17} \text{ GeV} \\
g^{1}_{\mu \nu}(x^0,\ldots,x^3), & \quad g^{1}_{\mu 4}(x^0,\ldots,x^3), \quad g^{1}_{44}(x^0,\ldots,x^3) & \sim 1 \times 10^{17} \text{ GeV} \\
g_{ij}(x^0,\ldots,x^4) & \Rightarrow g^{0}_{\mu \nu}(x^0,\ldots,x^3), \quad g^{0}_{\mu 4}(x^0,\ldots,x^3), \quad g^{0}_{44}(x^0,\ldots,x^3) & \sim 0 \text{ GeV}
\end{align*}
\]
What Brane-Worlds are...not yet

- Superstrings must have 10-dimensional spacetime
- 1984: Candelas-Horowitz-Strominger-Witten
  - *Kałuża-Klein* & supersymmetry ⇒ *Calabi-Yau* ($R_{\mu\nu} \approx 0$)
- 1986–87: Frenkel-Garland-Zuckerman, Rajeev-Bowick, Alvarez-Gaumé-Gomez-Reina, Pilch-Warner, Oh-Ramond, Harari-Hong: $R_{\mu\nu} \approx 0$ is a (super)string condition

- “Generic/typical” stringy spacetimes are:
  - Calabi-Yau (Ricci-flat: $R_{\mu\nu} \approx 0$, Wick-rotated complex 5-folds)
  - (Possibly, sectionally) non-compact
  - (Possibly) mildly singular
  - (Possibly) stratified
What Brane-Worlds are...

“Generic/typical” stringy spacetimes feature*:
- “isolated” 4D subspace(time)s;
- with matter localized to such subspace(time)s;

What Brane-Worlds are...

“Generic/typical” stringy spacetimes feature*:  
- “isolated” 4D subspace(time)s;  
- with matter localized to such subspace(time)s;  
- the first gedanken-prototype* of jump-gates and warp-drive: detouring into the “transversal” directions (hyper-space), then return  
- …and gravity?!  

What Brane-Worlds are...

1999: Lisa Randal & Raman Sundrum

Our spacetime is a sub-spacetime in a bigger (5-dimensional) one,

...for example, at its **boundary**

The “heterotic M-theory” of Hořava and Witten:

- two 10-dimensional boundaries in an 11-dimensional spacetime
- each with an $E_8$ gauge group
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  - two 10-dimensional boundaries in an 11-dimensional spacetime
  - each with an $E_8$ gauge group
- Two most cited papers in 5 years:
  - **RS-1**: exponential hierarchy (right)
  - **RS-2**: localized gravity (left)
    - ...but not both...
Localized matter (us)

The matter particles which constitute the observable Universe must be localized to this sub-spacetime:

Atiyah-Bott-Gårding-Candelas formula:

For algebraic varieties defined as the \( f(z) = 0 \) location \( Y \subset X \),

\[
\text{Res}_y[\cdots] := \frac{1}{2\pi i} \oint_{\gamma(Y)} \frac{dz}{f(z)} = \begin{cases} [\cdots]|_Y & \text{for } z \in Y \\ 0 & \text{for } z \notin Y \end{cases}
\]

...generalized* for all cohomology on \( Y \),

...in the supersymmetry limit (when C-analysis works)

Plus: a dynamical mechanism; see below

Localized Yang-Mills (interaction) fields (us)

EW & S: gauge quanta must be localized just like matter

And they are: in the \( A_\parallel - A_\perp \) decomposition,

...the former always have localized harmonic representatives

Localized fields (us)

- **Newtonian Gravity**: force must follow the $r^{-2}$-law
  - RS-mechanism:
    - $|x|$-like non-analytic functional dependence...
  - ...produces $\delta$-like terms in the Riemann curvature...
  - ...and $\delta$-like effective potential for fluctuations.
  - With a $-\text{sign}$, $\delta$-potential $\Rightarrow$ unique localized 0-mode!
  - The continuum “above” corrects Newton’s law: $G_N \left[ \frac{1}{r^2} + \frac{L^k}{r^{k+2}} \right]$
  - ...where $L$ is a curvature-related length scale.

**Mass-scale induction — a bonus!**

- Exponential hierarchy: re-scaling of $G_N$ and $M_P \sim 10^{19}$ GeV,
  - so that in the higher-dimensional spacetime $M_P \sim 10^{2}$ GeV;
  - This depends not on the extra dimensions’ size...
  - ...but curvature, near the brane-World.

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Brane-Worlds: an Introduction – continued

... in pictures:

If you can walk around it, it is codimension-2 or more.

Brane-Worlds as boundaries

Codimension = 1

Embedded brane-Worlds

Codimension > 1

Randall and Sundrum found it possible to: localize gravity in brane-World 1, and to obtain an exponential mass-hierarchy in brane-World 2 — but not together.
So, how does one localize gravity and induce a mass-hierarchy?

- Expand $g_{\mu\nu}(x, y) = \sum_k g^{(k)}_{\mu_x \nu_x} \phi_k(y)$ into “transversal” modes;

- prove the semi-infinity of the spectrum, $\{k \geq 0\}$;

- $g^{(0)}_{\mu_x \nu_x}$ is “our” localized graviton—iff $\phi_0(y)$ is bound to the brane-World;

- sum the contributions of $\phi_{k>0}(y)$ to Newton’s law:

\[
V(r) = M_D^{-D_B} \frac{m_1 m_2}{r_x} \sum_{k=0}^{\infty} \rho_k(r) \frac{|\phi_k(0)|^2}{\langle \phi_k | \phi_k \rangle} = G_N \frac{m_1 m_2}{r} \left[ 1 + \left( \frac{L}{r} \right)^n \right],
\]

so...

\[
G_N = M_D^{-D_B} \frac{|\phi_0(0)|^2}{\langle \phi_0 | \phi_0 \rangle} \quad & \quad \Delta V(r) = G_N \frac{m_1 m_2}{r} \frac{\langle \phi_0 | \phi_0 \rangle}{|\phi_0(0)|^2} \sum_{k>0}^{\infty} \rho_k(r) \frac{|\phi_k(0)|^2}{\langle \phi_k | \phi_k \rangle}.
\]

Here, $M_D = \sqrt{\frac{\hbar c}{G_N^{(D)}}}$ is the Planck mass in $D$ dimensions.
(Super)strings and Brane-Worlds

All (super)strings

- have a well-defined (point-field) limit, with (super)gravity,
- may be regarded as torus-compactifications of F-theory;
- so must contain:

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left( R - G_{\alpha\bar{\beta}} g^{\mu\nu} \partial_\mu \phi^\alpha \partial_\nu \phi^{\bar{\beta}} + \cdots \right),$$

where $2\kappa^2 = 16\pi G_N^{(D)}$ is the strength of gravity in $D$ dimensions.

- and the $\phi^\alpha$ are moduli fields (parametrizing the shapes and sizes of the spacetime)
Simplify*:
- consider a single modulus field, $\tau$, — for a torus,
- choose the appropriate (Teichmüller) metric,
- assume the spacetime metric to:
  - be of the “warped” direct product type,
  - depend only on a planar radius,
- assume $\tau$ to depend only on the planar angle:

$$d s^2 = A(z)\eta_{ab}dx^a dx^b + B(z)dz^2 + B(z)l^2d\theta^2, \quad z = \log(r/l)$$

$$\tau = \tau(\theta), \quad G_{\tau\bar{\tau}} = \frac{-1}{[\Im(m(\tau))]^2} \text{ (the Teichmüller metric)}$$

... and then solve the coupled metric-modulus (spacetime-matter) system.

*P. Berglund, D. Minic & T. Hübsch: see later for a complete bibliography.
The above simplification occurs only for matter that has no potential, which is true only of (superstrings’) moduli fields!

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}(\tau, \bar{\tau}), \quad g^{\mu\nu}[\nabla_\mu \partial_\nu \tau - \Gamma^\tau_{\tau\tau} \partial_\mu \tau \partial_\nu \tau] = 0, \]

but, \[ T_{\mu\nu}(\tau, \bar{\tau}) \overset{\text{def}}{=} \partial_\mu \tau \partial_\nu \bar{\tau} - \frac{1}{2} g_{\mu\nu} (g^{\rho\sigma} \partial_\rho \tau \partial_\sigma \bar{\tau}) \]

\[ R_{\mu\nu} = \tilde{T}_{\mu\nu} \overset{\text{def}}{=} \partial_\mu \tau \partial_\nu \bar{\tau}! \quad \text{Note: separation of variables!} \]

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\[ \tau'' + \frac{2}{\bar{\tau} - \tau} = 0 \]

is solved by:

\[ \tau_I(\theta) = a_0 + i g_s^{-1} e^{\omega(\theta - \theta_0)}, \quad a_0, g_s, \omega, \theta_0 = \text{const} \]

\[ \tau_{II}(\theta) = a_0 \pm g_s^{-1} \frac{\sinh[\omega(\theta - \theta_0)] + i}{\cosh[\omega(\theta - \theta_0)]}, \]
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}(\tau, \bar{\tau}) , \quad g^{\mu\nu} \left[ \nabla_\mu \partial_\nu \tau - \Gamma^\tau_{\tau\tau} \partial_\mu \tau \partial_\nu \tau \right] = 0 , \]

but,

\[ T_{\mu\nu}(\tau, \bar{\tau}) \overset{\text{def}}{=} \partial_\mu \tau \partial_\nu \bar{\tau} - \frac{1}{2} g_{\mu\nu} (g^{\rho\sigma} \partial_\rho \tau \partial_\sigma \bar{\tau}) \]

\[ R_{\mu\nu} = \tilde{T}_{\mu\nu} \overset{\text{def}}{=} \partial_\mu \tau \partial_\nu \bar{\tau} ! \quad \text{Note: separation of variables!} \]

\[ -\eta_{ab} \ell^{-2} \left[ a_0 \xi \text{sign}^2(z) Z^{\frac{D-1}{D-2}} e^{-\frac{\xi}{a_0} (\beta - Z^2)} - \varrho \delta(z) \right] = T_{ab} + T_{ab}^b , \]

\[ -a_0 \xi \text{sign}^2(z) = T_{zz} + T_{zz}^b , \]

\[ +a_0 \xi \text{sign}^2(z) + 2a_0 \delta(z) = T_{\theta\theta} + T_{\theta\theta}^b . \]

with \( \varrho := a_0 e^{-\frac{\xi}{a_0} (\beta - 1)} \left[ \frac{D-3}{D-2} - 2 \frac{\xi}{a_0} \right] \]

\[ ds^2 = A(z) \eta_{ab} dx^a dx^b + \ell^2 B(z) (dz^2 + d\theta^2) \quad Z(z) := 1 + a_0 |z| \]

\[ A(z) = Z^{\frac{2}{D-2}} \quad B(z) = Z^{-\frac{D-3}{D-2}} e^{\frac{\xi}{a_0} (\beta - Z^2)} \]
Neither of the two (only!) solutions for the matter fields are single-valued! Instead, they “jump” across the direction $\theta_0$:

$$\tau_1(\theta_0 + 2\pi) = \frac{a \tau_1(\theta_0) + b}{c \tau_1(\theta_0) + d}, \quad \tau_2(\theta_0 + 2\pi) = \tau_2(\theta_0) \pm n,$$

There is a 3-parameter space of choices for the first configuration, which for special choices of $a_0$, $g_s$ and $\omega$, contain integral solutions for $a,b,c,d$.

Both of these solutions exhibit an $SL(2,\mathbb{Z})$ monodromy, — just as superstring moduli (and no mere matter fields) do!

Thus, we have two configurations of a matter field which could not have stemmed from anything but superstrings.
The metric ("transversally")

\[ ds^2 = A(z) \eta_{ab} dx^a dx^b + B(z) dz^2 + B(z) l^2 d\theta^2, \quad z = \log(r/l) \]

\[ A(z) = Z \frac{2}{D-2}, \quad B(z) = Z^{-\frac{D-3}{D-2}} e^{\frac{\xi}{a_0} [\beta-Z^2]}, \quad Z(z) \overset{\text{def}}{=} 1 + a_0 |z| \]

With \( a_0 < 0 \), this metric exhibits naked singularities:
- where it (and an invariant measures of curvature!) grow unbounded;
- and which coincides with the horizon (place of no return for light);
- beyond which, the distance-squared is complex, i.e., unphysical.

With \( a_0 > 0 \), this metric exhibits time-like singularities:
- at \( z = \pm \infty (r = 0, \infty) \), time stops to advance.

In both cases, at \( z = 0 (r = l) \), there is a \( \delta \)-function like source.
The metric – in pictures

\( a_0 < 0 \), the \( \delta \)-function brane is sheathed by singularities

\( a_0 > 0 \), the \( \delta \)-function brane is coaxial with a singularity
Oh, by the way...

\[ \alpha_0 < 0, \text{ the } \delta\text{-function brane-World is sheathed by singularities; the yellow arrow is a traveler} \]

By moving off of the brane-World, the red traveler can move faster*, ahead of the Brane-World yellow

* A.F. Roane, *Some Brane-World Cosmological Models*, PhD dissertation
Bulk-roaming modes are unobservable... ...most of the time.

But, when they pass through “our” spacetime... ...and interact with us, ...they invalidate all our (local) conservation laws:

Thus we must:
- either ban all ET’s
- or localize them all!
A Dynamical “ET” Censoring Mechanism

- Matter localized to the \( \delta \)-function brane-World
  - with exact supersymmetry, may be represented by residues*;
  - upon supersymmetry breaking, become Gaussian distributions.

- Bulk-roaming matter (the “ET”’s)
  - represented by \( p \)-brane-probes of various spatial extension, \( p \);
  - the dynamics of which is “pseudo-relativistic”:

\[
S_p = - \tau_p \int d^{p+1} \xi \ e^{-\Phi} \sqrt{\det [G^s_{ab} + B_{ab} + 2\pi \alpha' F_{ab}]} + \mu_p \int d^{p+1} \xi \ C_{p+1},
\]

\[
[G^s_{ab}] = \text{diag}[-(A - Bv^2)e^{\Phi/2}, Ae^{\Phi/2}, \cdots, Ae^{\Phi/2}], \quad \text{set } B_{ab} = 0 = F_{ab}.
\]

Effectively, this is the dynamics of a relativistic particle:

\[ S_p = \int d^{p+1} \xi \mathcal{L}, \quad \mathcal{L} = -m c^2 \sqrt{1 - \frac{v^2}{c^2}} - \mathcal{V}, \]

\[ c = \frac{A(z)}{B(z)}, \quad m = \tau_p e \frac{(p-3)}{4} \Phi (A(z)B^2(z))^{p-1}, \quad \mathcal{V} = -\mu_p C_{p+1}. \]

As these vary radially, so does the brane-probe’s dynamics. Since

\[ \mathcal{E} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \mathcal{V} \quad \Rightarrow \quad v^2 = c^2 \left[ 1 - \frac{m^2 c^4}{(\mathcal{E} - \mathcal{V})^2} \right] \]

\[ \mathcal{E} \geq mc^2 + \mathcal{V} = \tau_p e \frac{(p-3)}{4} \Phi A^{p+1} - \mu_p C_{p+1} \]
However, this inequality pertains to the total, (Hamiltonian), *conserved* energy.

Yet, the brane-probes will radiate away their energy, through EM, gravitational, *etc.* waves.

But, without a detailed knowledge of the brane-probes’ interactions with other matter,

*How can one estimate the radiation loss?*

*How can one integrate its “deduction” from energy?*
Coupling to radiation—of any kind—modifies the momentum, and so also the velocity, to a “covariant” momentum:

\[ \vec{v} \rightarrow \vec{v} + c\vec{\Gamma} , \quad (\vec{v} + c\vec{\Gamma})^2 = v^2 + c^2\Gamma^2 , \quad \vec{v} \cdot \vec{\Gamma} = 0 . \]

This “connection” represents an average, combined connection, including all interactions of the brane-probe.

\[
\mathcal{E}' = \mathcal{V} + mc^2 A_{d-2} N \int_0^{\sqrt{1-v^2/c^2}} \frac{\Gamma^{d-3}d\Gamma}{\sqrt{1-v^2+c^2\Gamma^2/c^2}} ,
\]

\[ \mathcal{E}'|_{v=0} = \mathcal{E}|_{v=0} , \quad \mathcal{E}' = \mathcal{V} + mc^2 \left(1 - \frac{v^2}{c^2}\right)^{d-3/2} , \]

\[ \mathcal{E}'_{d=4} = \mathcal{V} + mc^2 \sqrt{1-v^2/c^2} , \text{ not conserved!} \]
The dynamics of $\mathcal{E}'$ being governed by \[ \frac{d\mathcal{E}'}{dt} = \{ \mathcal{E}, \mathcal{E}' \}_\text{PB}, \]

it is now possible to calculate:

- the total radiation loss,
- the rate of radiation loss,
- a life-time of the brane-probe’s motility.

This induces a dynamical mechanism for

- **banning** the brane-probes from the $\delta$-function brane-World when $a_0 < 0$, where the brane-World is sheathed by singularities,
- **localizing** the brane-probes to the $\delta$-function brane-World when $a_0 > 0$, where the brane-World is skirted by smooth space.

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-- a “latch-on, or be gone” mechanism.
Recall that $c = \frac{A(z)}{B(z)}$, $m = \tau_p e^{\left(\frac{p-3}{4}\right)} \Phi A^{p-1}(z) B^2(z)$, so $mc^2 + V = \tau_p e^{\left(\frac{p-3}{4}\right)} \Phi A^{p+1} - \mu_p C_{p+1}$ is an effective potential.
Recall that \( c = \frac{A(z)}{B(z)} \), \( m = \tau_p e^{\frac{(p-3)}{4}} \Phi A^{p-1}(z)B^2(z) \), so \( mc^2 + \mathcal{V} = \tau_p e^{\frac{(p-3)}{4}} \Phi A^{p+1} - \mu_p C_{p+1} \) is an effective potential.

The brane-probe is banned! The brane-probe is trapped!
Further reading

  (stringy cosmic strings, generalized later to stringy cosmic branes)

  (the “generic/typical” stringy spacetime, jump-gates, warp-drives, etc.)

- ———, *JHEP* 01 (2001) 045
- ———, *JHEP* 02 (2001) 010
- ———, “work in progress” (some unfinished business)

  (physics, Howard University: a gedanken prototype of hyperspace jump-
The End

or, more likely,

...to be continued...